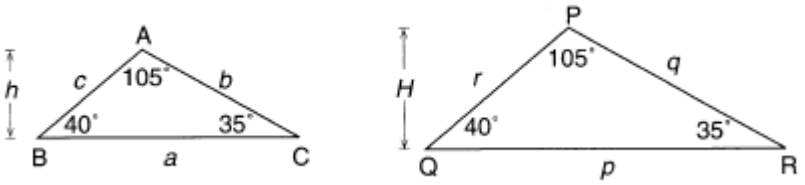
Similar triangles

Two polygons are said to be **similar** if (a) they are equi-angular, and (b) corresponding sides are in proportion. For triangles (a) ⇔ (b).

In the diagrams Δ ABC and Δ PQR are similar.



For similar figures the ratios of the lengths of the sides are the same and represent the **scale factor**, i.e.

$$\frac{p}{a} = \frac{q}{b} = \frac{r}{c} = k \text{ (where } k \text{ is the scale factor of enlargement)}$$

The heights of similar triangles are proportional also:

$$\frac{H}{h} = \frac{p}{a} = \frac{q}{b} = \frac{r}{c} = k$$

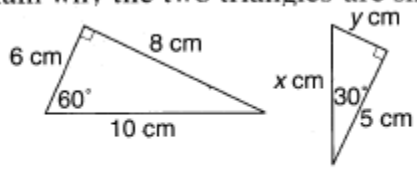
The ratio of the areas of similar triangles (the **area factor**) is equal to the square of the scale factor.

$$\frac{\text{Area of } \Delta \text{ PQR}}{\text{Area of } \Delta \text{ ABC}} = \frac{\frac{1}{2}H \times p}{\frac{1}{2}h \times a} = \frac{H}{h} \times \frac{p}{a} = k^2$$

$$\frac{\text{Area of } \Delta \text{ PQR}}{\text{Area of } \Delta \text{ ABC}} = \left(\frac{H}{h}\right)^2 = \left(\frac{p}{a}\right)^2 = \left(\frac{q}{b}\right)^2 = \left(\frac{r}{c}\right)^2 = k^2$$

Exercise 1

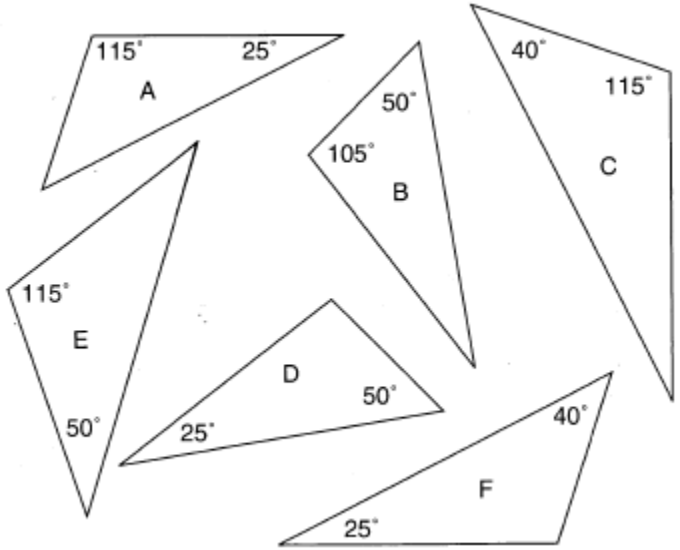
1. a) Explain why the two triangles are similar.



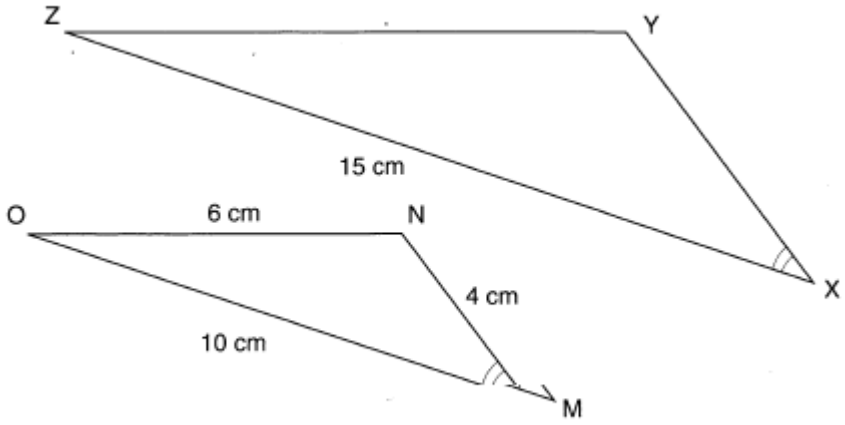
b) Calculate the scale factor which reduces the larger triangle to the smaller one.

c) Calculate the value of x and the value of y.

2. Which of the triangles below are similar?

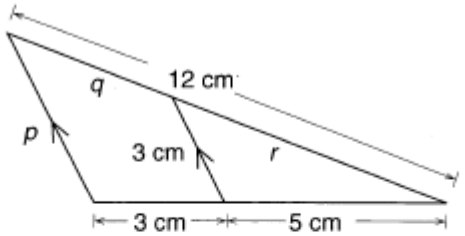


3. The triangles below are similar.

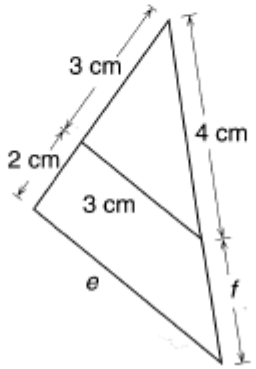


a) Calculate the length XY.
b) Calculate the length YZ.

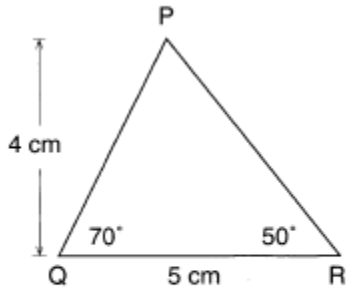
4. In the triangle below calculate the lengths of sides p, q and r.

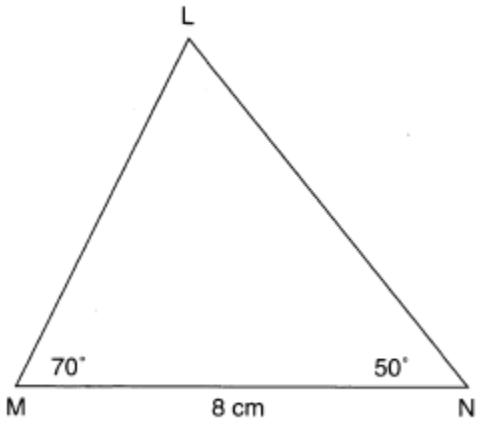


5. In the triangle below calculate the lengths of sides e and f.



6. The triangles PQR and LMN are similar.





Calculate:
a) the area of Δ PQR,
b) the scale factor of enlargement,
c) the area of Δ LMN.

7. A square is enlarged by increasing the length of its sides by 10%. If the length of its sides was originally 6 cm, calculate the area of the enlarged square.

8. A square of side length 4 cm is enlarged by increasing the lengths of its sides by 25% and then increasing them by a further 50%. Calculate the area of the final square.

9. An equilateral triangle has an area of 25 cm^2. If the lengths of its sides are reduced by 15%, calculate the area of the reduced triangle.