1) Upper and Lower bounds of a Measurement

The simple rule is this:

The real value can be as much as HALF THE ROUNDED UNIT above and below the rounded-off value.

1) A room is given as being '9 m long to the nearest METRE' — its actual length could be anything from 8.5 m up to 9.5 m — i.e. HALF A METRE either side of 9 m. So 8.5m and 9.5m are the lower and upper bounds.

2) If it was given as '9.4 m, to the nearest 0.2 m', then it could be anything from 9.3 m up to 9.5 m (9.4 m ± 0.1 m) — i.e. 0.1 m either side of 9.4 m. So 9.3 m and 9.5 m are the lower and upper bounds.

3) If a length is given as 2.4 m to the nearest 0.1 m, the rounded unit is 0.1 m so the real value could be anything up to 2.4 m ± 0.05 m giving answers of 2.45 m and 2.35 m for the upper and lower bounds.

4) 'A school has 460 pupils to 2 Sig Fig' (i.e. to the nearest 10) — the actual figure could be anything from 455 up to 464. — (Why isn't it 465?) So 455 and 464 are the upper and lower bounds.

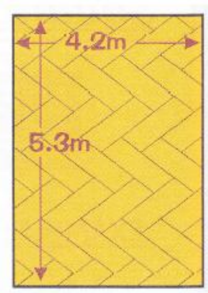
2) Maximum and Minimum Values for Calculations

When a calculation is done using rounded-off values there will be a DISCREPANCY between the CALCULATED VALUE and the ACTUAL VALUE:

EXAMPLE: A floor is measured as being 5.3 m × 4.2 m to the nearest 10 cm.
Calculate the minimum and maximum values for the area and perimeter.

Multiplying 5.3 m by 4.2 m gives an area of 22.26 m^2, but this is not the actual floor area because the real length and width values could be anything from 5.25 m to 5.35 m and 4.15 m to 4.25 m,

∴ Maximum possible floor area = 5.35 × 4.25 = 22.7375 m^2,
∴ Minimum possible floor area = 5.25 × 4.15 = 21.7875 m^2.
Also, using these values:
Maximum possible perimeter = (5.35 + 4.25) × 2 = 19.2 m,
Minimum possible floor area = (5.25 + 4.15) × 2 = 18.8 m.



Exercises

1) A yacht is described as 17 metres long to the nearest 0.1 m. What is the longest and shortest it could be?
2) x and y are measured as 2.32 m and 0.45 m to the nearest 0.01 m.
a) Find the upper and lower bounds of x and y.
b) If $z = x + 1/y$, find the max and min possible values of z.

Careful here — the biggest input values don't always give the biggest result.